Re/Os constraint on the time-variability of the fine-structure constant

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Abstract

We argue that the accuracy by which the isochron parameters of the decay $^{187}\text{Re} \rightarrow {}^{187}\text{Os}$ are determined by dating iron meteorites may not directly constrain the possible time-dependence of the decay rate and hence of the fine-structure constant α. From this point of view, some of the attempts to analyze the Oklo constraint and the results of the QSO absorption lines are re-examined.

Recently it has been claimed [1] that the long-lived beta decay $^{187}\text{Re} \rightarrow {}^{187}\text{Os}$ should constrain the time-variability of the fine-structure constant α more strongly than pointed out before [2]. At the core of the argument, however, the authors assumed that the limit of time variation of the decay rate is of the same order as the accuracy by which the decay rate itself is determined. One of the purposes of this note is to re-examine this hypothesis by checking the basic procedure of dating meteorites. We find that the decay rate measured at the *present time* is a *time-average* of the *local* decay rate which can be time-dependent. Obviously the rate of change, which is also time-dependent in general, is entirely different from the accuracy of the average. In spite of this difference in principle, there might be a correlation in practice, which is, however, to be tested observationally.

In order to show how we have reached this conclusion, we begin with a brief account of how the decay rate of rhenium can be used to date iron meteorites for which recent analysis was carried out with an improved precision [3]. The fundamental formulation will then be followed by the comments, (i) the implication of direct measurement of today's value of the decay rate, (ii) how Dyson's original idea is affected by our present knowledge of dating, and (iii) a theoretical example in which the uncertainty of the averaged decay rate comes from the time dependence of the decay rate, illustrating that the relation between them can be complicated even if there is any in practice.

We also emphasize that the present issue attracts wide attention because of the question if what is claimed to be the constraint from dating meteorites can be understood consistently with other constraints on the time-dependence of α, from the Oklo phenomenon [4] and the QSO absorption lines [5,6]. For this reason, the second half of this note is devoted to a phenomenological re-examination of the theoretical analyses in terms of the cosmological scalar field [7–9], in which fitting all of the observations on the changing α has been attempted. We find that the result is rather disappointing if we accept an important feature of the analysis of the QSO data; a rather "flat" distribution of $\Delta\alpha/\alpha$ [5,6]. We then suggest that an even better fit can be obtained if we do not impose the constraint from Re-Os decay.

Suppose a meteorite is formed at the time $t = t_1$, which is about 4.6 Gys ago, and will be called the "meteorite time" in what follows. After this time, the amounts of ^{187}Re and the related ^{187}Os

are conveniently measured in terms of the ratios to the stable isotope ^{188}Os: $N_{\mathrm{Re}} \equiv {}^{187}\mathrm{Re}/^{188}\mathrm{Os}$ and $N_{\mathrm{Os}} \equiv {}^{187}\mathrm{Os}/^{188}\mathrm{Os}$, respectively. The former would decay according to

$$N_{\mathrm{Re}}(t) = Ae^{-\lambda(t-t_1)}, \tag{1}$$

with the decay rate λ assumed to be constant, for the moment. In practice, we consider a group of meteorites supposed to have started nearly at the same time, $t = t_1$. The initial value $A = N_{\mathrm{Re}}(t_1)$ may be different from meteorite to meteorite due to chemical fractionations under different initial pressures and temperatures. For each meteorite, the amount lost would appear as ^{187}Os;

$$N_{\mathrm{Os}}(t) = A\left(1 - e^{-\lambda(t-t_1)}\right) + B, \tag{2}$$

where B denotes the amount which had been present in the solar system before t_1 not necessarily due to the decay of ^{187}Re. It seems reasonable to assume B which is common throughout the group. By eliminating A from (1) and (2) we obtain

$$N_{\mathrm{Os}}(t) = S(t)N_{\mathrm{Re}}(t) + B, \tag{3}$$

where the time-dependent $S(t)$ is given by

$$S(t) = e^{\lambda(t-t_1)} - 1, \tag{4}$$

which starts with zero, increasing as $t - t_1$.

At the present time t_0 with $t_0 - t_1 \approx 4.6 \times 10^9$y, we measure $N_{\mathrm{Re}}(t_0)$ and $N_{\mathrm{Os}}(t_0)$, plotting the latter against the former which takes different values because of different A. The result should be a straight line (3) with the common slope $S(t_0)$ given by (4) with t_0 for t, and the intercept B. This plot is called the Re-Os "isochron," as shown in Fig. 1 of [3]. If the age $t_0 - t_1$ is determined by other more accurate isochrons, U-Pb, for example, we may determine the rate λ.

Recently four groups (IIA, IIIA, IVA and IVB) of iron meteorites have been analyzed carefully [3], determining the slopes with the accuracy of 0.5%. It was argued [1] that this accuracy is to be the same as the one by which $\Delta\lambda/\lambda$ of the time-variation of λ is constrained. They also exploited the theoretical relation [2]

$$\frac{\Delta\lambda}{\lambda} \approx -1.9 \times 10^4 \frac{\Delta\alpha}{\alpha}, \tag{5}$$

where the "amplification factor" on the right-hand side comes from an exceptionally small Q-value, which makes the decay of ^{187}Re particularly suitable to probe possible time-variability of α in the Coulomb-only approximation. Imposing the same 0.5% for the accuracy in determining the constant above on the left-hand side now interpreted as the one for the possible time-variation yields

$$\left|\frac{\Delta\alpha}{\alpha}\right| \lesssim 2.5 \times 10^{-7}, \tag{6}$$

over the time span of $t_0 - t_1$. As it turns out this is roughly as strong as the constraint obtained from the Oklo phenomenon, and is nearly two orders of magnitude more stringent than the nonzero value

2

indicated by the QSO result. We will show, however, that the left-hand side of (5) may not be 0.5% mentioned above.

Suppose now λ varies with time. Since the law of decay is described originally in terms of a differential equation, (1) is modified to

$$N_{\mathrm{Re}}(t) = A \exp \left(\int_{t_1}^{t} \lambda(t')dt' \right). \tag{7}$$

The expression (1) with $t = t_0$ holds true if λ is re-interpreted as an average:

$$\bar{\lambda} = \frac{1}{t_0 - t_1} \int_{t_1}^{t_0} \lambda(t')dt'. \tag{8}$$

All we can do from dating meteorites at the present time is to measure the average (8), which masks any variation during the whole history between t_1 and t_0. In this respect the method is different in principle from [4] and [5,6], in which the *past events* are directly probed. We have no way to distinguish potentially different functions $\lambda(t)$ as long as they give the same average value. Any time-dependence in $\lambda(t)$ is allowed no matter how precisely $\bar{\lambda}$ is determined. We are going to illustrate the points by a few examples.

In order to detect time-dependence of λ, one of the most direct ways in principle is to find the present value λ_0, though it ought to be a difficult experiment because of an extremely small rate. A recent accurate measurement [10] provides

$$\lambda_0 = (1.68 \pm 0.50) \times 10^{-11} \mathrm{y}^{-1}, \tag{9}$$

which is consistent with the value of $\bar{\lambda} = 1.67 \times 10^{-11} \mathrm{y}^{-1}$ used in [3]. From the uncertainty of 3% in (9) follows that $\lambda(t)$ might have deviated from $\bar{\lambda}$ *at least* by 3%, or even by 6%, sometime during the whole history of the meteorite, though depending on what time interval defines $\Delta\lambda/\lambda$.

We might assume the simplest linear approximation in which we have a straight line for $\lambda(t)$ which passes through $\bar{\lambda}$ at the middle, $(t_1 + t_0)/2$, giving $|\Delta\lambda/\lambda| \sim 3 \times 10^{-2}$ at $t = t_1$. Combining this with (5) we find that $|\Delta\alpha/\alpha| \approx 1.5 \times 10^{-6}$ toward 4.6 Gys ago. This turns out to be much larger than (6). Notice that we could change the slope, the rate of time-change of λ without changing the average, as long as we keep the middle point intact. A more precise determination of λ_0 will lower the lower-bound, but will never lower the upper-bound.

As a historical example of a non-linear function, let us re-evaluate Dyson's proposal [2] who argued that if α toward the meteorite time was very large, and so was λ, we would find no leftover $^{187}\mathrm{Re}$ around us today. The contradiction can be avoided if a possible change of $\lambda(t)$, and of $\alpha(t)$, near the meteorite time is kept below certain limit.

Dyson's idea can be represented by a simplified step function,

$$\lambda(t) = \left\{ \begin{array}{ll} \lambda_2, & \text{for} \quad t_1 \leq t \leq t_2, \\ 0, & \text{for} \quad t_2 \leq t \leq t_0, \end{array} \right. \tag{10}$$

where t_2 is an intermediate time between t_1 and t_0 while the constant λ_2 is chosen to be higher than $\bar{\lambda}$, so that the decay life-time defined by $\tau_2 = \lambda_2^{-1}$ is shorter than $\bar{\tau} = \bar{\lambda}^{-1}$ (by 200 times in [2]).

3

In order to allow the largest possible λ_2 for a given interval $t_2 - t_1$, we chose in the second line the minimized $\lambda(t)$ satisfying the natural condition $\lambda(t) \geq 0$. We also add another narrow region prior to t_0 for which $\lambda(t) = \bar{\lambda}$, assuming that today's λ equals $\bar{\lambda}$. The time interval of this region can be chosen very small, however, such that its effect to the integral will be negligibly small.

By requiring that the integral of (10) gives $\bar{\lambda}$ correctly, we obtain the relation

$$(t_2 - t_1)\lambda_2 = (t_0 - t_1)\bar{\lambda}, \quad \text{or} \quad \frac{t_2 - t_1}{\tau_2} = \frac{t_0 - t_1}{\bar{\tau}}. \tag{11}$$

By noticing that $\bar{\tau} = 6 \times 10^{10}$y is much larger than $t_0 - t_1$ in the last equation, we conclude

$$t_2 - t_1 \ll \tau_2, \tag{12}$$

implying that there was no time sufficiently long for the Re decay to be completed. In other words, we still see rhenium around us no matter how short its decay life-time might be at the earliest time. This shows that the established $\bar{\lambda}$ no longer allows a naive argument [2].

As another exercise of the technique of a step function, let us start with a constant $\lambda(t) = \bar{\lambda}$ then introduce a local change $\lambda(t) = \lambda_2 \equiv \bar{\lambda} + \delta$ in the interval $t_1 \leq t \leq t_2$. We are replacing the second line, 0, of (10) by $\bar{\lambda}$. The relative time-change $\Delta\lambda/\lambda$ may be defined by the change of λ from "today's" value $\bar{\lambda}$ divided by certain average of $\lambda(t)$. The former is obviously δ while the latter is close to $\bar{\lambda}$ as long as $|\delta| \ll \bar{\lambda}$, hence

$$\frac{\Delta\lambda}{\lambda} = \frac{\delta}{\bar{\lambda}}. \tag{13}$$

The uncertainty of the average $\bar{\lambda}$ will be determined by something different, like the isochron parameters. It still seems interesting to suppose that possible inaccuracy in evaluating the contribution from δ causes $\Delta\bar{\lambda} \sim ((t_2 - t_1)/(t_0 - t_1))\delta$. By combining this with (13) we find

$$\frac{\Delta\bar{\lambda}}{\bar{\lambda}} \sim \frac{t_2 - t_1}{t_0 - t_1} \frac{\Delta\lambda}{\lambda}. \tag{14}$$

This illustrates how the two kinds of relative changes can be different from each other even if they share the same common origin, δ in the range $t_1 \leq t \leq t_2$. In contrast to a *naive* expectation for the basic equality between the two, as expressed in [1], we realize that the relation depends on how $\lambda(t)$ behaves, through the ratio $(t_2 - t_1)/(t_0 - t_1)$ in our simplified example. We also point out that the numerator $t_2 - t_1$ on the right-hand side of (14) is simply the interval of a nonzero change of $\lambda(t)$, independently of when it occurred, not necessarily at the earliest time. A question might then be raised against a procedure in [7–9], in which a constraint (6) on $\Delta\alpha/\alpha$ is imposed right on the meteorite time.

We also note that, apart from the precise timing, the relation (14), substituted from (5) on the right-hand side, together with the isochron constraint $|\Delta\bar{\lambda}/\bar{\lambda}| \lesssim 0.5\%$ may even derive $|\Delta\alpha/\alpha| \sim 0.5 \times 10^{-5}$, nearly the same as the weighted mean of the QSO result [5,6], if we choose $(t_2 - t_1)/(t_0 - t_1) \sim 0.05$.

We have so far argued conceptually that the constraint (6) may not be derived unambiguously, particularly at or near the meteorite time. Now as a test of the "hypothesis" in [1], we are going to show purely phenomenologically that this constraint is likely in conflict with the available QSO result.

For the later convenience, we define the fractional look-back time $u = 1 - t/t_0$. We then recall that the data points in [6] (the "fiducial sample," Fig. 8) distribute from $u = 0.203(z = 0.229)$ to $u = 0.875(z = 3.666)$. The meteorite time $u_{\mathrm{met}} = 0.33$ is above the lowest data point, but certainly far below most of other data points. All the 128 data points are summarized by

$$y \equiv \frac{\Delta\alpha}{\alpha} \times 10^5 = -0.54 \pm 0.12, \tag{15}$$

with $\chi^2_{\mathrm{rd}} = 1.06$. Accommodating (6), or $|y| = 0.025$, into the overall fit $y(u)$ might be achieved either by i) an exponential fall as u decreases, or by ii) an oscillatory $y(u)$ with one of zeros nearly identified with u_{met}. The former choice has an advantage that other two important constraints, the Oklo constraint at $u_{\mathrm{oklo}} \approx 0.14$ and $y(0) = 0$ (by definition) would be complied almost automatically.

We thus start with assuming

$$y(u) = ae^{b(u-u_{\mathrm{met}})}, \tag{16}$$

where $b > 0$ (u is in the opposite direction with t) while $a = y(u_{\mathrm{met}})$. We expect that this is a behavior that simulates the dominant pattern of the fits attempted in [7–9] reasonably well in the relevant time range.

We determine a and b by minimizing χ^2 for the QSO data. The reduced chi-squared $\chi^2_{\mathrm{rd}} = \chi^2/(128 - 2) = 1.07$ is obtained for $a = -0.354$ and $b = 1.39$. The above χ^2_{rd} turns out to be nearly the same as 1.06 for the simple weighted mean (fit in terms of a horizontal straight line, $y = -0.54$), while the size a turns out more than an order of magnitude larger than the desired value 0.025 for (6). The corresponding fit is shown in Fig. 1, in which only the 13 binned data (see Fig. 8 of [6]) is shown for a easier comparison with the fit.
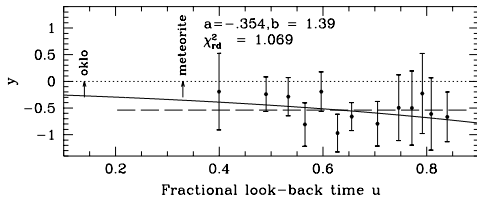

Figure 1: An exponential fit (16) to the QSO result [6] shown as a binned fit, though the minimizing χ^2 itself has been made with respect to the original 128 data points. The minimum $\chi^2_{\mathrm{rd}} = 1.069$ is obtained for $a = -0.354$ and $b = 1.39$. The horizontal long-dashed line represents the weighted mean $y = -0.54$. A flat distribution of the data is obvious. The Oklo time $u_{\mathrm{oklo}} = 0.14$ and the meteorite time $u_{\mathrm{met}} = 0.33$ are shown by upward arrows.
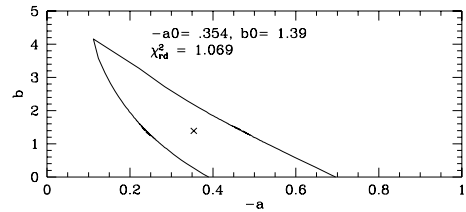
Figure 2: The 68% confidence region around the minimized χ^2 of the fit corresponding to Fig. 1, as indicated by a cross.

We also estimated the confidence region for 68%, as shown in Fig. 2, which shows that $a = -0.025$ is entirely out of the contour. Around this value of a, we find no 2-dimensional minimum of χ^2. We tentatively fixed $a = -0.025$ and searched for b to minimize χ^2, obtaining $b = 7.28$ with $\chi^2_{\mathrm{rd}} = 1.13$.

By comparing the corresponding fit shown in Fig. 3 with the previous Fig. 1, we find an obvious reason for a poorer agreement; the assumed small $-y(u_{\mathrm{met}})$ required a sufficiently large value of b, which in turn forced $-y$ to rise too much as u increases, thus enhancing χ^2. In other words, a flat distribution of the QSO data points, as was seen also in the previous result [5], dose not favor a small $-y(u_{\mathrm{met}})$. The same consequence appears to be shared by the fits in [7] (with $E = 12$), in [8] (Fig. 3) and in [9] (Fig. 13).
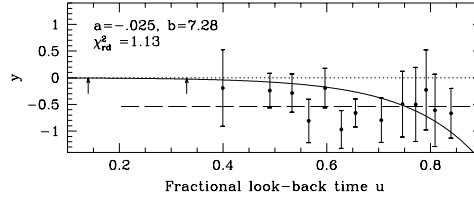

Figure 3: A tentative fit with a fixed $a = -0.025$ but with $b = 7.28$ that minimizes chi-squared, resulting in $\chi^2_{\mathrm{rd}} = 1.13$. Notice too sharp a fall of the curve toward the higher u.

We have further tried to see tentatively if the situation improves by replacing $u_{\mathrm{met}} \approx 0.33$ by $u_{\mathrm{oklo}} \approx 0.14$, but finding little changed basically. We admit that the exponential suppression alone, like (16), fails to accommodate the QSO result, as long as we accept the detailed behavior shown by [5,6].

We have developed, on the other hand, a fit based on the damped-oscillator behavior as inspired by the scalar field in the scalar-tensor theory of gravity playing a crucial role in understanding the accelerating universe [11]. In [12] we assumed a phenomenological function

$$y = ae^{bu}\sin\left(2\pi\left(u - u_{\mathrm{oklo}}\right)/T\right), \tag{17}$$

which is chosen to *vanish* at u_{oklo} as an approximate expression for a small value $|y(u_{\mathrm{oklo}})| = 10^{-2} - 10^{-3}$. The smallest $\chi^2_{\mathrm{rd}} = 1.09$ was obtained for $a = 0.151, b = 2.4, T = 0.714$, fitting the QSO result nearly as good as the weighted-mean with $\chi^2_{\mathrm{rd}} = 1.06$ [6]. Notice that b is smaller than 7.28 as was used in Fig. 3. This made it easier to be consistent with the flat behavior of the QSO data toward the high-u end. Recall that we did not appeal to a strong suppression of the exponential function for the expected small value of $|y(u_{\mathrm{oklo}})|$.

We then find it difficult, however, to satisfy the constraint (6) at u_{met} as well, because $u_{\mathrm{met}} - u_{\mathrm{oklo}} \approx 0.19$ is much smaller than T, which is sufficiently large to keep the data distribution "flat." Alternatively we may choose u_{met} as one of the zeros of y and then trying to suppress $y(u_{\mathrm{oklo}})$ sufficiently small. This cannot be immediately ruled out as long as we accept b as large as 8. This choice, apart from the tendency of the cosmological solutions favoring smaller b, makes it more difficult to comply another natural condition $y(0) = 0$. If we follow the foregoing discussion *not* to appreciate the constraint (6) at the meteorite time, we may propose a better, though still phenomenological, fit to satisfy both of the constraints at $u = 0$ and u_{oklo} to be comfortably consistent with the QSO result,

6

as will be shown separately.

We finally point out that the constraints still from the Oklo and QSO results might be related to the evolution of the scalar field affecting directly the way of cosmological acceleration, through which we further look into the possibly changing α in the eras of CMB and the nucleosynthesis [11].

One of us (Y.F.) thanks Hisayoshi Yurimoto, Department of Earth and Planetary Sciences, Tokyo Institute of Technology, for his help to understand physics of meteorite dating.

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